Synergetics. An interdisciplinary approach to self-organization in complex systems

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Abstract: Synergetics is an interdisciplinary field of research that deals with complex systems composed of many individual parts. The systems and their parts may belong to quite different disciplines, ranging from physics over medicine till epistemology. Some examples are: physics: fluids, gases, plasmas composed of atoms, molecules, ions, chemistry: reactions of molecules, forming patterns, biology: morphogenesis (cells), evolution (species, animals), sociology: humans, ecology: humans, animals, plants, neuroscience: brain composed of neurons. Synergetics focuses its attention on those situations where new qualities appear on macroscopic scales, e.g. new spatio-temporal patterns in fluids, new percepts in brain a.s.o. The emergence of new qualities can be treated theoretically by means of concepts such as instability, order parameters, slaving principle, circular causality, which will be explained in detail. Their mathematical/algorithmic basis will be outlined stressing the interplay between nonlinearity and randomness. All the systems considered are in interaction with their surround which imposes special conditions (modeled by control parameters) on the systems.

1. Goal

In view of the wide scope of synergetics which may appeal to a wide audience I am trying to present the following contribution in a pedagogical style. We will consider complex systems that occur in a variety of disciplines, such as physics, chemistry, biology, and medicine, but also sociology, economy and ecology. In all the cases considered, the systems are composed of many components, subsystems, elements, or parts.

In the following we will speak of parts or components. They interact with each other by exchange of energy, matter and/or information. This enables them to produce spatial, temporal, or functional structures. Let us consider some typical examples. In physics, fluids and gases are composed of atoms or molecules that form quite different movement patterns (e.g. (Fig.4)). We may observe cloud streets with our own eyes. In chemistry, reacting molecules may form macroscopic patterns such as stripes or propagating rings. In biology, cells may form stripe- or spot-patterns on skins (fish) or furs.

Neuroscience deals with the human brain which is the most complex system we know. It consists of about 100 billion neurons, where a single neuron can be connected with up to 10000 other neurons. This "system" enables our recognition of faces and objects, movement patterns, it serves movement control of our limbs, it produces our thoughts and allows us to express them by speech and gestures, it homes our feelings, just to mention a few characteristic features. When considering animals as parts of a system, biology abounds with examples: evolution of species, flocks of birds, swarms of fish, spread of diseases. Movement science studies coordination of limbs of humans or animals (the term "Synergy" was coined by Sherrington for the coordination of muscles!). In sociology the parts are represented by humans showing specific collective behaviors including formation of public
opinion or even making revolutions. Robotics devises the joint action in multi-robot systems. There are numerous further examples for structure formation (including coordinated actions) in ecology, economy, and even epistemology. What have all these systems (and their parts) in common? They produce their structures without specific interference from the outside, in *contradistinction* to the *action of a sculptor*. In other words, they achieve their structures (in a wide sense) via self-organization. The basic question of synergetics is: *Are there general principles of selforganization irrespective of the nature of the parts?* Over the past 45 years, this question could be answered in the positive for large classes of systems. In my article I will illucidate the most important concepts by means of simple examples and indicate briefly the mathematics behind them. The key strategy of our approach is to focus our attention on those situations where the *macroscopic structure* of a system changes *qualitatively*. I illustrate such situations by two examples from physics which will allow me to introduce basic concepts of synergetics.

![Diagram of gas atoms and mirrors](image)

Fig. 1. *Upper part:* Typical set up of a gas laser. A glass tube is filled with gas atoms and two mirrors are mounted at its end faces. The gas atoms are excited by an electric discharge. Through one of the semi-reflecting mirrors, the laser light is emitted. *Lower part, left:* In a conventional lamp, the light field generated by the gas atoms consists of individual wave trains that represent noise. *Right:* In the laser, a highly ordered, i.e. coherent, light wave is generated.

2. *The Laser Paradigm or Boats on a Lake*

The light source laser (Fig. 1) produces a unique kind of light, namely coherent light. Let us consider the example of a gas laser. A glass tube contains a gas of laser-active atoms. By means of an electric current sent through the glass tube, the individual atoms (or molecules) are energetically excited. After excitation, they can act like a miniature radio antenna, but emitting light waves instead of the radio waves of conventional antennas. If the rate of excitation is increased, more and more light waves can be emitted. They are reflected by the mirrors, which serve to trap the light wave in the laser device for some time. Thus the concentration of light waves grows. Beyond a critical density of light waves, a new process becomes decisive, one that was first introduced by *Einstein*. Namely, when a light wave hits
an excited atom, it may force the atom to give away its energy to exactly this wave so that the light wave is enhanced. This process is called stimulated emission. The enhanced light wave may hit a further excited atom and quite obviously in this way a light avalanche may start. Different kinds of light waves may be emitted. A competition between the growing waves sets in and is, eventually, won by one of them which may use the energy of the excited atoms most efficiently. The amplitude of this wave is called the order parameter. We remind the reader that in physics the light field is represented by its electric field strength. This wave with its order parameter “enslaves” the individual gas atoms in the following way: Each atom may be considered as being composed of a nucleus and one specific electron orbiting around it. As is well-known, electric fields act on charged particles such as electrons. When the light wave oscillates, it acts in an oscillating fashion on the electron and forces it to move in phase. This process can be visualized as follows: Consider a lake on which boats are floating. An electric field may be put in analogy to a water wave and the individual electrons to the boats. When the water wave propagates across the lake, the boats will be pulled up and down according to the wave. Similarly, electrons move according to the oscillations of the electric field strength. This is the slaving principle in action. Under the influence of the electric field, the electrons start an oscillation. On the other hand, it is known from electrodynamics that oscillating charges, in our case oscillating electrons, generate an electric field. Thus while the order parameter (jointly with its wave) enslaves the electrons (atoms, parts), and determines their behavior, the electrons (parts) generate the order parameter. This is the circular causality principle. The behavior of the order parameter can be visualized as follows. Consider a hilly landscape (Fig. 2), first with only one valley with grassy slopes on both sides. A stone (or ball) on a slope will slide down to the bottom of the valley. Now identify the horizontal displacement of the ball (cf. Fig. 2) with the size of the amplitude of the light field (i.e. the order parameter). Clearly, even if the stone is pushed upwards, it will return to its resting position. This situation corresponds to one in the laser where the atoms emit randomly light waves (“fluctuations”) but the electric excitation current is too weak to start laser action.

Fig. 2. Visualisation of an order parameter $\xi$.
Upper part: control parameter $\alpha < \alpha_c$ (critical), middle part: $\alpha = \alpha_c$, lower part: $\alpha > \alpha_c$.
R.h.s. order parameter versus control parameter: bifurcation
When the strength of the electric current is enhanced, the resulting situation is described by a very flat bottom. Kicks at the stone will push it far away from its equilibrium position to which it will slide back only slowly. In the jargon of phase transition theory we are dealing with “critical fluctuations” and “critical slowing down” of the order parameter. When the excitation current is enhanced still further, a qualitatively new situation arises (Fig. 2, lower part). The two new valleys offer the order parameter two different stable equilibrium positions which are resistant to fluctuations (sleep slopes!), enabling a stable light field amplitude, i.e. a highly ordered state (cf. Fig. 1). Which of the two valleys is “occupied” by the order parameter is determined by an initial fluctuation, i.e. a chance event! (For the physicists: my picture is oversimplified because “in reality” there is a ring valley (like a Mexican hat)).

3. A simple example from fluid dynamic

![Diagram](image)

Fig. 3. By heating a fluid from below and cooling from above, a temperature difference \( T_1 - T_2 \) may be generated

![Diagram](image)

Fig. 4. Beyond a critical temperature difference, a fluid may develop a roll pattern

When a control parameter \( \alpha \) (in that case the temperature difference between the lower and upper surface) is changed beyond a critical value, the system suddenly forms a new macroscopic state that is quite different from the formerly homogeneous state in which the fluid was at rest. Let us look more closely at the example of the fluid in which a roll patterns forms. Note that this well-known example from fluid dynamics merely serves for an illustration of basic concepts of synergetics and cannot replace the full theory. We start with a control parameter value \( \alpha_0 \), where the state of the system is known. We denote this state by \( q_0 \). In the case of a fluid, the state \( q_0 \) is the resting state. Then we change the control parameter until suddenly a new state occurs. After the control parameter has been changed, the fluid starts its motion, or, in other words, the rotation speed of the rolls increases in the course of time. This increase indicates an instability: The old resting state has become unstable. The newly developing motion grows out of a small fluctuation, such as the density fluctuations of the molecules of the fluid or its local velocity fluctuations. The first step of an analysis consists in a study of the behaviour slightly above the instability point. As it turns out, the system may undergo quite different collective motions, a few examples
of which are shown in Fig. 5 (left-hand side). The velocity (Fig. 6) of some of these configurations tends to grow, whereas others decay even if they have been initially triggered by some fluctuation (Fig. 5, right-hand side). The amplitude $\xi$ of the growing configuration is of particular interest for the further analysis, because it determines the evolving macroscopic patterns, for instance, a roll pattern or other patterns. When pattern formation starts, the amplitudes are initially small. This allows us to study the growth and decay of these individual configurations independently of each other. When the amplitudes grow further, the configurations start to influence each other. For instance, they may compete with each other so that only one wins the competition and suppresses all the other configurations. In other cases, they may coexist or they may even stabilize each other. The amplitudes of the growing configurations are the order parameters. They describe the macroscopic order, or, more generally speaking, the macroscopic structure of the system. The state $q$ of the system can be described by a superposition of all configurations, i.e., the growing and the decaying ones. If a system has many components, there are correspondingly many individual configurations. That means the information needed to describe the behavior of the system is not reduced by a decomposition into its configurations. But now a central
theorem of synergetics comes in. It tells us that the total space-time behavior of the state \( q \) is governed (or enslaved) by the order parameters. This is again the *slaving principle* of synergetics (Fig. 7). Since, in general, the number of order parameters is much smaller than the number of components of a system, in this way we achieve an enormous reduction of the degrees of freedom, or in other words, an enormous information compression takes place. In a way, the order parameters act as puppeteers that make the puppets dance. There is, however, an important difference between this naive picture of puppeteers and what is happening in reality. As it turns out, by their collective action the individual parts, or puppets, themselves act on the order parameters, i.e., on the puppeteers. While on the other hand the puppeteers (order parameters) determine the motion of the individual parts, the individual parts in turn determine the action of the order parameters (*circular causality*). The principle of circular causality allows us to interpret the slaving principle in yet another fashion. Because the individual parts of the system determine or even generate the order parameters which in turn enslave the individual parts, the latter determine their behavior *cooperatively*. It is tempting to describe this phenomenon in anthropomorphic terms as *consensus finding* by the individual parts. Thus enslavement and *consensus finding* are two sides of the same coin. This insight is crucial when we apply the concepts of synergetics to *social processes*.

![Diagram](image)

Fig. 7. Visualization of the slaving principle.
One or several order parameters enslave the behavior of the subsystems described by the variables \( q_1, q_2, \ldots \).

4. *Instability/structure-hierarchies*

What happens when a control parameter is increased further and further? Does a newly established structure become more stable? Both the laser and the fluid examples provide us with an important insight: While first the structure (coherent wave, roll pattern) becomes more stable against perturbations from the outside, at some further increased control parameter value \( \alpha_1 \) (in the fluid: a critical temperature difference between the lower and upper surface) the rolls start to oscillate (like a string) with one frequency. At the next \( \alpha_2 \), two frequencies appear, while at a still longer \( \alpha_3 \) an irregular ("chaotic") roll motion sets in. Instability hierarchies are found in quite a number of systems.

5. *A glance at general conceptualization and steps towards a mathematical approach*

We distinguish the components by an index \( j = 1, \ldots, N \), where \( N \) may be a very large number (Fig. 8) and denote the activity of the component \( j \) by \( q_j \). Such activity may, for instance, be the firing rate of a neuron, but there are many other interpretations of \( q_j \) depending on the kind of system. In order to characterize the activity of all the parts of a system, we must list the individual components. It is convenient to combine them into the state vector

\[
q = (q_1, q_2, \ldots, q_n).
\]
In a number of cases, the components may be considered as being continuously distributed, for instance, we may treat a fluid as a continuum. In such systems we shall replace the index \( j \) by the spatial coordinate \( x, y, z = x \) and

![Diagram](image)

Fig. 8. Examples of systems described by the variables \( q_1, q_2, \ldots \). Left: A network with nodes whose activities are described by these variables. Right: An arrangement of cells, for instance in a tissue, where the state of each cell with index \( j \) is characterized by a variable \( q_j \). Such a variable may refer, for instance, to the concentration of a particular chemical (e.g. a pigment) in such a cell.

\[ q_j \] by \( q(x) \). An example of \( q(x) \) is provided by the density of molecules in a fluid where the density varies as a function of the spatial coordinate \( x \). In many cases, the state of a system will change in the course of time, i.e., the state vector \( q(t) \) becomes a function of time \( t \)

\[ q = q(t) \] \hspace{2cm} (2)

In order to study this temporal change in synergetics, we adopt the following attitude: We assume that the temporal change of \( q \) may be determined by a number of factors, namely

1) by the present state \( q \) of the system
2) by the connections between the components \( q_j \)
3) by control parameters \( \alpha \) and
4) by chance events.

Let us discuss these different factors in more detail.

5.1 Present state. A very simple example is provided by the overdamped motion of a point mass fixed to a spring. There is a certain equilibrium position of the mass. When we elongate the spring and let the point mass move, under usual conditions it will undergo an oscillatory process. When we suppose, however, that the whole process is going on, say, in some kind of molasses, the friction may be so strong that no oscillations can occur. In such a case, we obtain overdamped motion. If we denote the deviation from the equilibrium position by \( q \) the equation of motion can be cast into the form

\[ v = -kq \] \hspace{2cm} (3)

where \( v = \frac{dq}{dt} \), velocity, and \( k \) is a constant. Clearly, the temporal change of \( q \) (left-hand side of (3)) depends on its present state, namely \( q \) (right-hand side of (3)).
5.2 Links between the \( q_j \). An example is provided here by a spring that connects two point masses (1) and (2) with positions \( q_1 \) and \( q_2 \), respectively. If \( a \) is the equilibrium distance between the point masses, then the force acting on point mass 2, is given by

\[
F = -k(q_2 - q_1 - a),
\]

(4)

where \( k \) is the spring constant.

This example shows that two quite different quantities enter in the equations of motion, namely on the one hand the variables \( q_j \) and, on the other hand, specific constants, such as \( k \) and \( a \). This also holds in far more complicated cases, for instance, the links can be synaptic strengths between two neurons. In continuously distributed media, such as fluids, or chemical distributions, where we deal with space-dependent concentrations \( q(x) \) of molecules, the links may have the form of a gradient or may depend on still higher spatial derivatives of \( q(x) \).

5.3 Control parameters. In Sect. 3 we considered a fluid in a vessel that is heated from below and cooled at its upper surface (Fig. 3). Because of the cooling and heating, a temperature difference between the lower and upper surfaces, \( \alpha = T_1 - T_2 \), will be established. If the temperature difference \( \alpha \) exceeds that critical value, a macroscopic motion of the fluid becomes visible. We may say that the macroscopic behavior of the system is controlled by the control parameter \( \alpha \). Similar dramatic changes may be observed in chemical reactions. When, in a chemical reactor to which chemicals are continuously added and removed by an overflow, the concentration of some chemical is increased beyond a critical value, an oscillation may suddenly set in. The chemicals may change their colour, say, from red to blue to red, and so on. In this case, the concentration of the added chemical serves as a control parameter. Even in the most complex system, namely the brain, we may identify control parameters. These may be the concentrations of neurotransmitters, such as serotonin or dopamine, or the concentrations of administered drugs, such as Haloperidol, caffeine, etc. (For instance, when we drink coffee we may become more active). In a number of cases the concentrations of hormones may also be considered as control parameters.

Some caution must be used, however, when we apply the concept of control parameters to biological and some other systems. In physical and chemical systems, we fix the value of the control parameter(s) by imposing experimental conditions from the outside, for instance, by the amount of heating in the case of a fluid. In biological systems the control parameters are quite often produced by the system itself and are thus in a way variables. But these variables change slowly compared to the actions they trigger.

5.4 Chance events. Finally, we have to discuss chance events or random events. Many fields of science are dominated by the idea that there are no chance events, but rather that all processes are entirely deterministic. There are, however, a number of important processes which involve chance events. They are abundant in quantum physics, which deals with the behavior of atoms and molecules. An example is provided by the spontaneous emission of light by atoms. When an atom is excited, it may emit a light wave, or, in quantum mechanical terms, a photon, but we are unable to predict the emission time. Precisely speaking, all elementary processes in chemical reactions are of quantum mechanical and, therefore, of a random nature. According to our understanding of physics, these chance events are fundamental and cannot be predicted by the development of a more detailed theory.

There is yet another kind of randomness, namely that of thermal fluctuations, e.g., density fluctuations in gases, fluids, or solids, (or electric current fluctuations in semiconductors and metals). Such fluctuations occur even if we neglect the quantum nature of molecules, i.e., if we treat them as classical particles. In this case we are seemingly dealing with chance events
(or fluctuations) because of our lack of knowledge of the precise positions of the molecules. Thus, except in quantum theory, the question of whether we should speak of chance events or not may depend on our level of description. For instance, we may treat the motion of gas atoms according to classical mechanics, i.e., according to a fully deterministic theory. Nevertheless, we may treat the fluctuations of density as if they were chance events that can be described by statistical theories. There is good reason to believe that chance events also occur in our brain (leading e.g. to unexpected behavior). Such events may be the spontaneous opening of vesicles in neurons, or the random firing of neurons, or the occurrence of tremor. But with our present state of knowledge of the important microscopic processes in the brain, it is not clear whether their fluctuations are of a fundamental, quantum mechanical nature, or merely depend on our level of description. Chance events play a fundamental role, especially close to instabilities (cf. Sect. 2). In many cases they trigger a new state and even determine which one is selected out of several possible ones (cf. Fig. 2). So far we have discussed the factors that influence the temporal development of the state (vector) \( q \). The corresponding eqs. (cf. also the appendix) refer to numerous systems and may be very complicated. Now we present two general concepts of synergetics which were exemplified in Sects. 2 and 3, i.e. slaving principle and order parameters.

6. *The Slaving Principle*

The relationship between the behavior of the individual parts of a system and the order parameters, i.e. the slaving principle, can be given a general and abstract form. It does not matter whether the parts are discrete or continuously distributed. Let us again denote the variable (or a set of variables) that describes the part \( j \) by \( q_j \) and consider several order parameters \( \xi_1, \xi_2, \ldots, \xi_M \). Then the slaving principle states

\[
q_j = f_j(\xi_1, \ldots, \xi_M).
\]

Or in words: the state \( q_j \) of part \( j \) is uniquely determined by the order parameters. In particular this holds true if the order parameters are time dependent. We shall call a configuration of a system that is described by a state vector of the form \( q = (q_1, \ldots, q_N) \) with \( q_j \) given by (5) a *collective mode*, or, for short, a mode. Since, in general, the number of order parameters \( M \) is much smaller than the number of the parts of a system, (5) is a mathematical expression of the *information compression* mentioned above. At first sight, the illustration of the slaving principle in the cases of the fluid and of the laser seems to be rather different, because in the first case we have been speaking about configurations or collective motions of the fluid whereas in the case of the laser we have been considering the motion of individual electrons in the laser atoms. However, mathematical analysis, which we shall not dwell on here, shows that both descriptions are equivalent and depend on the kind of problem.

Both cases have a fundamental common feature, namely in both cases we can clearly distinguish between the role of order parameters and of the enslaved variables or components. When we perturb the order parameters and then drop the perturbation, the order parameters relax only slowly. When we perturb the individual parts, they relax quickly, i.e., order parameters and parts (components) are distinguished by their *time scales* If we wish to apply the principles of synergetics to the brain or any other system, e.g. in ecology or sociology, we must thus ask whether the time-scale separation holds.
7. The Central Role of Order Parameters

Let us summarize our results. We consider a class of systems that have the following properties: When one, or maybe several, control parameters are changed, the system enters an instability. In other words, it leaves its former state and starts to form a qualitatively new macroscopic state. Close to the instability point, different kinds of collective configurations occur; some of them grow, whereas others decay after their generation by fluctuations. By a study of the growing and decaying states, we may distinguish between the unstable and stable configurations and are thus led to the configurations which are governed by the order parameters. The order parameters determine the behavior of the individual parts via the slaving principle. Thus the behavior of complex systems can be described and understood in terms of order parameters. At the same time, we need no longer consider the action of behavior of the individual parts, but may instead describe the total system by means of the order parameters. The slaving principle underlying this relationship thus leads to an enormous information compression. In quite a lot of cases, the number of order parameters is very small and we may discuss the behavior of the system in terms of these order parameters. For instance, in the laser the number of atoms, may be, say $10^{18}$, whereas the number of order parameters is just one, namely the electric field strength of the winning mode. In addition we may state that via this order parameter the motion of the individual electrons becomes highly correlated. Similarly, in the brain we are dealing with myriads of neurons, but with much fewer behavioral patterns (which are still numerous, however!). Order parameters are abstract quantities. In many cases, they acquire their specific meaning via the slaving principle.

One warning should be added at the end of this section. Because the principles of synergetics have been illustrated by means of examples from physics, namely liquids and lasers, one may prematurely draw the conclusion that synergetics represents a physicalism. This, however, is not true at all, because in synergetics we start from abstract mathematical relationships, which then are applied to numerous systems including those of physics. But because physical systems are still comparatively simple as compared to biological systems, they provide the nicest way to illustrate the meaning of the mathematical principles of synergetics.

We have discussed the contents of the slaving principle more or less qualitatively. Readers who are interested in the mathematical proof of the slaving principle (which is actually a theorem) are referred to my book Synergetics (see References). The proof requires that the system considered is close to an instability point. General arguments as well as numerous practical applications of the slaving principle have shown, however, that in quite a number of cases it also holds in systems farther away from their instability points. For further information on these, see the works listed in “Further Reading”.


Our previous discussion clearly shows what we understand by self-organization of a system. Namely, quite generally, when certain external or internal control parameters are changed, there are certain situations where the system itself does not change slightly, but undergoes a dramatic change of its macroscopic state as is witnessed by the spontaneous formation of patterns in lasers and liquids. It is superfluous to say that biology and medicine abound with such qualitative changes that range here from the formation of spatial patterns or structures (morphogenesis) to dramatic changes in behavior, for instance, in psychosis. It is important to note that the control parameters do not anticipate the evolving macroscopic patterns. For instance, a liquid is heated quite uniformly; nevertheless the interaction between its molecules produces quite distinct patterns.
The spontaneous formation of patterns by self-organization seemed to be in conflict with the second law of thermodynamics. According to this law, in so-called closed systems macroscopic order should disappear and be replaced by a homogeneous state that may, however, show a chaotic motion at its microscopic level. For instance, the motions of gas atoms are quite chaotic, but at the macroscopic level a gas appears to be practically uniform. This tendency towards a maximally chaotic state at the microscopic level and a structureless state at the macroscopic level is formulated by the statement that the entropy of a system increases to its maximum value. However, this statement only holds for closed systems which are not driven by input of energy and matter from their surroundings. The biologist von Bertalanffy recognized that biological systems are open systems, whose structures and functions are maintained by an influx of energy and matter either in the form of sunlight and material as used by plants, or in the form of nutrients and oxygen as used by animals. Von Bertalanffy coined the term Fliessgleichgewicht (flux equilibrium) to characterize this state of living matter. All systems treated in synergetics may be considered as open systems and thus fulfill a necessary condition for self-organization.

9. Examples from neuroscience
Psychophysics and order parameters

Our starting point is a typical relation between order parameters and the enslaved parts: While order parameters react to external influences ("perturbations") slowly, parts act on a faster time-scale (time-scale separation). This invites us to the following analogy with brain processes: While percepts are processed on time scales of 1/10 sec or still longer, neurons function on a time scale of milliseconds. These facts suggest to establish an analogy.

percepts ↔ order parameters
neurons ↔ parts (elements)

Note the ontological question that lurks behind this analogy!

Nevertheless, let us study a few typical cases of order parameter dynamics with respect to perception. A typical order parameter potential has two valleys indicating two different stable order parameter values, i.e. bistability. Which is actually happening in perception (Fig. 9). Do you perceive a vase or two faces? Thus, the same picture "induces" two quite different percepts, i.e. "bistability" in perception. Strictly speaking over a somewhat longer time span, oscillations happen. In fact, in the case of two order parameters, oscillations may occur (limit cycles in the sense of dynamical system theory). The perception dynamics was mathematically modelled under the assumption that each percept is controlled by an "attention" parameter that fades away after that percept is recognized. As I learned later, Gestaltpsychologist Wolfgang Köhler had made the same suggestion in 1920 (though he didn't model it mathematically). Our model allowed us to establish several relationships between first recognition time,
Fig. 9. Bistability in perception.

Fig. 10. Oscillations in perception. $\xi_1, \xi_2$ are order parameters, representing percepts (e.g. vase/faces).

Fig. 11. Hysteresis. The position of the ball in a local minimum of a potential $V(\xi)$ depends on history. Upper part from left to right: When the potential is changed the ball is still in the left minimum in the middle part of this fig. Lower part from right to left: The ball is now in the right valley.
bias, recognition times etc. and to make contact with experimental results (cf. Fig. 10). A further example is hysteresis. Hysteresis means that the state of a system depends on history (Fig. 11). Fig. 12 provides us with an example from perception: The switching from the perception of a man's face (upper left corner) to that of a kneeling woman (lower right corner) depends on the sequence in which we look at this series of pictures.

Fig. 12. Hysteresis in perception (cf. text)

10. Pattern recognition by the synergetic computer as model of perception

Here, I exploit an analogy between pattern formation and pattern recognition. This analogy is based on the concepts of order parameters, on the slaving principle and on circular causality (For a review cf. (Haken 1990)). In pattern formation, let initially a part of the total system be in an ordered state. This part calls on, in general, several order parameters which then compete among each other. The initially stronger order parameter wins this competition ("principle of winner takes all") and, eventually enslaves the total system, i.e. it establishes a fully ordered pattern. In pattern recognition, the individual parts are features, e.g. grey values of pixels into which a pattern is decomposed. Consider as a concrete example face recognition (Fig. 13, 14). Then only some features, e.g. that of a nose, may be given. Those features call upon order parameters which compete among each other, the initially strongest wins and, again via the slaving principle, restores the whole pattern, e.g. a face. The recognition process (Fig. 14) is based on the following algorithm, which I formulate, quite in the spirit of Synergetics, both at the microscopic (feature) level and at the macroscopic (order parameter) level.

At the microscopic level, each pixel \( l, l = 1, \ldots, L \), is represented by its grey value \( q_l \), which is mapped onto a neutral net so that \( q_l \) is also the excitation level of the model neuron \( l \). Then I introduced evolution equations for the state vector \( \sigma = (q_1, \ldots, q_L) \),

\[
\dot{q}(t) = -\nabla_q V(q, c).
\]

(2)

where the potential \( V \) is a polynomial of \( q \) up to fourth order with coefficients that can be interpreted as synaptic strengths. \( V \) describes a hilly landscape which I constructed in such a way that each of its valleys corresponds to one and only one of the prototype patterns: The
corresponding values of the coefficients can be either inserted “by hand” into the computer or, more importantly, learned by the rule

$$\langle V(q,c) \rangle_q = \min!$$  \hspace{1cm} (3)

where the average $\langle \cdot \rangle$ refers to a sequence of partially incomplete patterns whose “idealization” is thus achieved. My algorithm was implemented by my former co-worker Armin Fuchs on a serial computer (cf. Figs. 13, 14) where recognition has been made invariant against displacements, rotation and scaling. Using attention parameters and their fading away (cf. Sect. 9) our approach was also able to recognize faces in a complex scene. The transition to the macroscopic (order parameter) level is achieved by the transformation of the pixel vector $q$

$$q(t) = \sum_k \xi_k(t) v_k + \text{rest}, \hspace{0.5cm} k = 1, \ldots, k \leq L$$  \hspace{1cm} (4)

where $\xi_k(t)$ is the order parameter associated with the prototype pattern vector

$$v_k = (v_{k1}, \ldots, v_{kL}).$$  \hspace{1cm} (5)

The resulting order parameter equations are

$$\dot{\xi}_k = \xi_k \left( \lambda_k + a \xi_k^2 - b \sum_m \xi_m^2 \right).$$  \hspace{1cm} (6)

$\lambda_k \geq 0$ attention parameters, $a,b > 0$.

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**Fig. 13.** Example of stored prototype patterns (after Fuchs, Haken (1988))

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**Fig. 14.** Example of recognition process (after Fuchs, Haken (1988))
11. Back to the phenomenological level: Psychology and psychotherapy

Interestingly, Synergetics, originally quite unexpected, has made its way into psychology and psychotherapy. Clearly in the present context, a few remarks must suffice here.

Behavioral patterns may be conceived as order parameters. Thus changes of behavioral patterns can be interpreted as phase transitions, often with their typical features, e.g. critical fluctuations. Research in Synergetics has revealed the important role of the principle of indirect steering. This means, the change of a control parameter can induce the evolution of a new (behavioral) pattern by means of self-organization. This has fired a discussion on appropriate control parameters in psychotherapy: specific verbal interventions, or specific drugs? Or both? Eventually, Synergetics cannot escape to try an answer to the eternal mind-body problem. My suggestion is the analogy

\[
\text{body} \leftrightarrow \text{parts} \\
\text{mind} \leftrightarrow \text{order parameters}
\]

Thus in view of the principle of “circular causality” mind and body are just two sides of the same coin. As I had learned in the meantime, this was just the opinion of Spinoza.


Beneficial and disastrous control parameters

As Synergetics has shown by means of a comprehensive theory, and as is witnessed by numerous observations, control parameters play a fundamental role in selforganization of complex systems. Their change may lead to new structures. But looking at them from a different point of view: their change can also destroy a (desired) structure. Close to critical control parameter values, a complex system is highly sensitive even to minute changes of the control parameters, and can change its state dramatically. A few hints may suffice here. Both local and global climate may act as control parameter. E.g. in mountains, local climate parameters change with height. While at sea level there may be jungle, at the mountain top no vegetation at all, and inbetween clearly distinct zones of vegetation – and correspondingly – animal populations.

A simple example of ecology: When the pollution of a lake is increased more and more its fish populations does not decrease proportional to the degree of pollution, but it disappears entirely beyond a critical amount of pollution. Clearly, literature on ecology and climatology abounds with theories and observations. The great difficulty in making predictions rests on the precise calculation of the critical control parameters, in which specific models and often unsufficiently known parameters and variables enter. Nevertheless, Synergetics provides us with an important though qualitative insight. When specific control parameters come close to critical values (critical points), critical fluctuations and critical slowing down happen i.e. abnormal values of typical measurable quantities, e.g. temperature, rain falls, floods, tornados etc. Clearly, such critical phenomena are important warning signals which must be taken very seriously. Besides the material critical control parameters, and still more fundamental, there is an immaterial parameter: The general public attitude towards the conservation of climate and natural resources. This is mandatory in view of our uncertainty how close we are to the material critical values. In spite of being immaterial this control parameter can be measured quantitatively by polls. And it can be established as an order parameter representing selforganized public opinion. In this context self-organization means: People form their opinion by learning the consequences on their own lives and of others, if the critical value is surpassed.
13. Sociology: mechanism of revolutions

Synergetics allows us to get some insight into quite a number of sociological processes, such as formation of public opinion, competition and cooperation among groups of people etc. Here I mention an extreme example which shows how we may discuss the “mechanism” of revolutions from a systemic and systematic point of view:

Step 1: Destabilization of a system: bad social conditions, terror activities, …

Step 2: Critical fluctuations, large demonstrations, outbreaks of violence, …
   Critical slowing down: relaxation times for return to normal become longer and longer.

Step 3: beyond instability (no more relaxing outbreak of revolution) several options for new ordered states. A small group of people (chance events?) may decide where to go now.

After I had deduced this scenario from our Synergetics’ results, I learned that these three steps had been described already by Lenin.

14. Concluding remarks and outlook

In my contribution I have tried to elucidate the mechanism of selforganization based on the cooperation of the individual parts of a system. In this way remarkably large classes of systems can be treated in terms of general concepts, i.e. in particular order parameters and slaving principle. Besides a verbal description, in quite a number of cases these concepts can be put on a concrete mathematical basis. For sake of clarity I have discussed a “microscopic” approach starting from the behavior of the individual parts (e.g. the laser atoms) described by variables $q_I$. Another approach is based on making “unbiased estimates” on complex systems on which only a limited amount of data is available and which is based on information theory (cf. “Further reading”). Out of the vast field of Synergetics with its relations to many scientific disciplines, I have presented a small section. One example was a brief sketch of our attempts to model some aspects of brain function to elucidate how basic concepts of Synergetics can be applied to this fascinating field. As our studies (seem to) suggest, the human brain manages to compress the complexity of perception and action time and again into low dimensional dynamics of a rather small number of – in each case appropriately established – order parameters. Critics may object that this is a too narrow view. On the other hand, our brain manages to compress the complexity of our world all the time: e.g. by categorization as witnessed by language. Thus I think that the Synergetics approach may be a useful tool to cut one’s way through the jungle of the brain’s complexity. At any rate, this issue brings me to discuss the relation between Synergetics and the presently flourishing field of Complexity Science. Synergetics is surely one (or even the) forerunner of Complexity Science, both of which share their emphasis on interdisciplinarity. But there are also differences that are best explained by looking at the different styles of scientific work:

1. production of new data (information production)
2. formulation of principles, laws etc. (information compression)

When I defined the scope of Synergetics I strongly emphasized 2. Searching for common principles still remains an important goal which has to go along also with 1.
Appendix

Sketch of the mathematical approach

The dynamics of the system’s components is described by stochastic nonlinear partial differential equations of the form

\[
\frac{dq(x,t)}{dt} = N(\alpha, q, \nabla) + F(\alpha, q, \nabla, t)
\]

(1)

where \( q \in R^M \), \( x \in R^L, \) \( L = 1, 2, 3 \) \( \alpha \) : set of control parameters, \( \nabla \) : nabla operator acting on \( q \). \( N \) : deterministic part, \( F \) : stochastic part defined by its correlation functions in time \( t \).

\( q \) : subject to initial and boundary conditions.

Approach

For control parameter set \( \alpha = \alpha_0 \), solution \( q \) to (1) with \( F \equiv 0 \) be known \( q = q_0(x,t) \).

- a) \( q_0 \) time-independent, stable fixed point
- b) \( q_0 \) time-periodic, stable limit cycle
- c) \( q_0 \) quasi-periodic, stable torus.

Upon change of \( \alpha \), a stability analysis allows us to transform the original equations into those for the order parameters \( \xi_u \) (characterized by positive Lyapunov exponents in linearization) and phases \( \Phi_v \), and those for the enslaved mode amplitudes \( \xi_s \) (negative Lyapunov exponent in linearization) and their couplings, i.e.

\[
\frac{d\xi_u}{dt} = N_u(\xi_u, \xi_s, \Phi_v, \nabla) + F_u
\]

(2)

\[
\frac{d\Phi_v}{dt} = N_v(\xi_u, \xi_s, \Phi_v, \nabla) + F_v
\]

(3)

\[
\frac{d\xi_s}{dt} = N_s(\xi_u, \xi_s, \Phi_v, \nabla) + F_s
\]

(4)

in an obvious representation. The algorithm of the slaving principle allows us to eliminate \( \xi_s \) from (2) – (4) in two steps

1) We may express \( \xi_s \) as functional of \( \xi_u, \Phi_v \)

\[
\xi_s(x,t) = \mathcal{F}_s(\xi_u(x,t), \Phi_v(x,t), t)
\]

(5)

by an explicit procedure, where the explicit time-dependence of \( \mathcal{F} \) stems solely from the fluctuating forces. In practice \( \mathcal{F} \) can be approximated by low power polynomials of \( \xi_u \). (2) is a general form of the slaving principle (where contact can be made with central manifold and inertial manifold theorems).
2) We insert (5) into (2), (3) and obtain a closed set of equations for the order parameters. In a number of cases, the solutions of these equations can be classified, e.g. potential case, limit cycles, deterministic chaos.

An explicit example of my approach is the derivation of Ginzburg-Landau type equations. To study the system very close to instabilities, Fokker-Planck equations have been used.

*Further Reading (containing numerous references)*


